Estimation of the Time-to-Go Parameter for Air-to-Air Missiles

Gordon K.F. Lee*
Colorado State University, Fort Collins, Colorado

The design of optimal controllers for air-to-air missiles has been of much recent interest. Many control laws require a good estimate of the time-to-go (tgo) before intercept. This paper presents an algorithm for estimating time-to-go using the current relative position and velocity characteristics of the missile and target. By constructing an objective function representing teminal miss distance, one is able to obtain an iterative scheme for estimating tgo. Simulation studies using a smart target algorithm and this methodology have yielded good results that show improvement over conventional algorithms for estimating tgo. A comparative investigation of the methodology developed in this paper with other algorithms is presented.

I. Introduction

THE air-to-air intercept problem can be formulated in many ways. The primary objective of this problem is to minimize the terminal miss distance (hit the target) under unknown target maneuvers. Secondary objectives include using minimal control effort over the entire scenario, minimizing time, and minimizing the relative positions between the missile and target over all time. Overall constraints in developing methodologies that satisfy such design objectives should be real-time or near-real-time computation and easy implementation.

Several models have been proposed[1-4] for the air-to-air scenario. This paper concentrates on a linearized model based upon aerodynamic properties. With the air-to-air models so chosen, the control objective is to choose a missile command to eventually hit a maneuvering target.

The traditional approach to choosing such a missile command is proportional navigation. While this method is relatively easy to implement and works well for simple scenarios, proportional navigation may result in less than acceptable trajectories for complex flights.[5]

Several other guidance laws have been proposed[5]; many of these approaches require more information than proportional navigation. In some cases, a good knowledge of the time-to-go parameter is required; the purpose of this paper is to develop a possible method to estimate time-to-go that, when used in some of these newer guidance laws, can produce more desirable results than proportional navigation under complex air-to-air missile scenarios.

Section II presents the basic linearized system model and the optimal guidance control law. Some of the assumptions are then removed resulting in a suboptimal guidance law. A necessary parameter in implementing such a guidance law is the time-to-go (tgo) before intercept. Section III develops the algorithm for estimating tgo. A comparison of this methodology with other formulations through simulation results is given in Section IV, while the derivation of the estimation of tgo is developed in the Appendix.

The methodology developed has the appealing characteristics of being computationally attractive to implement as well as comparable (and better in certain flight maneuvers) to existing tgo estimates. A further attractive feature of this methodology is that, because it is based upon optimal control theory, the algorithm gives some insight into the effects of various performance indices on the overall terminal miss-distance objectives.

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*Associate Professor, Department of Electrical Engineering.

II. Problem Development

Before developing the linearized optimal control law, the notation used in this paper is: ( ) denotes a column vector, $(\ )^T$ its transpose, I the identity matrix, and $(\ )^{-1}$ a matrix inverse.

Consider the following dynamics representing a linearized time-invariant air-to-air scenario:

$$\dot{x}(t) = \begin{bmatrix} 0 & I \\ 0 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ I \end{bmatrix} u(t) = Ax(t) + Bu(t) \qquad (1)$$

where

$$x(t) = \begin{bmatrix} x_p(t) \\ x_v(t) \end{bmatrix}$$

represents the augmented relative position vector $x_p(t)$ and relative velocity vector $x_v(t)$ in the x, y, and z missile body coordinates (see Fig. 1). $u(t)$ represents the commanded acceleration control vector. In representing the air-to-air scenario by Eq. (1), it is assumed the $u(t)$ has no constraints. This is in fact not true and the physical constraints associated with the allowable control will be presented later.

Suppose it is desired to hit the target with a minimal control effort. A representative cost functional for this performance

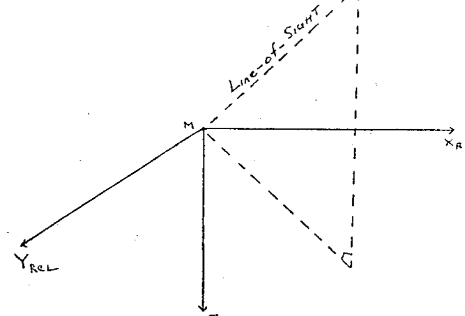

Fig. 1 Missile and target using relative coordinates.

objective may be[5]

$$J = x^T(t_f) S_f x(t_f) + \frac{1}{2}\int_{t_0}^{t_f} u^T(t) R u(t)\,\mathrm{d}t$$

where

$$S_f = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \quad R = \mathrm{diag}(r) \tag{2}$$

where t_0 is the initial time, t_f the final time, and $\mathrm{diag}(\cdot)$ a diagonal matrix with a nonzero element r.

Direct optimization of Eq. (2) with respect to the control vector $u(t)$ can be accomplished by constructing the Hamiltonian,[6]

$$H = \tfrac{1}{2} u^T(t) R u(t) + \lambda^T(t)\,[Ax(t) + Bu(t)] \tag{3}$$

By applying the maximum principle on H, one finds the optimal control law is given by

$$u^*(t) = -R^{-1} B^T \lambda(t) \tag{4}$$

where $\lambda(t)$ is the costate vector. By assuming t_f, t_0, $x(t_0)$, and $\lambda(t_f)$ are given, the optimal control law is usually solved via a two-point boundary value method (which requires integration forward and backward in time). If t_f is unknown (which is the case for the air-to-air problem), a term due to the transversality condition on unspecified final time is added to Eq. (3).

Under the assumption that t_f is given or can be accurately estimated, due to the simplicity of R, B, A, and S_f, a direct solution can be obtained to Eq. (3). Since the primary objective is to hit the target, one can let $r \to 0$ in Eq. (2), resulting in a Mayer-type problem. Following Riggs,[5] this results in

$$u^*(t) = -Kx(t) \tag{5}$$

where

$$K = (3/\mathrm{tgo}^2)\,[I \;\; \mathrm{tgo} I] \tag{6}$$

and $\mathrm{tgo} = t_f - t =$ time-to-go before intercept. Equation (5) represents an optimal control law if tgo is known and if $u(t)$ has no restrictions (i.e., the target acceleration is known or indentically zero). As $u(t)$ is restricted to an allowable subspace for physical reasons, Eq. (5) must be modified. This law can be further improved if an accurate estimate of time-to-go is obtained. Inclusion of estimates of target acceleration is an area of future study.

III. Estimation of Time-to-Go

In order to estimate time-to-go, one may optimize Eq. (2) with respect to tgo. However, this results in much computation. For analysis purposes, it is desirable to estimate tgo using the following design objective:

$$J_{\mathrm{tgo}} = x^T(t_f) S_f x(t_f) + q\left(\int_0^{\mathrm{tgo}} \mathrm{d}\tau\right)^2 \tag{7}$$

where q is a weighting factor on time. Note that Eq. (7) represents a performance criterion based upon the terminal miss distance and time-to-go. Optimization of Eq. (7) with respect to tgo results in an estimate of time-to-go that can then be used in Eq. (6).

To minimize Eq. (7) with respect to tgo, we note that

$$x(t_f) = e^{\bar{A}(t_f - t)} x(t) + \int_t^{t_f} e^{\bar{A}(t_f - \tau)} Bc(\tau)\,\mathrm{d}\tau \tag{8}$$

where $\bar{A}$ represents the closed-loop system matrix using an appropriate feedback control law and $c(t)$ represents a disturbance input. We noted earlier that Eq. (5) assumed no restrictions on $u(t)$. As present air-to-air missiles do not have a commanded control in the x direction, a modification of Eq. (5) is necessary.

Let

$$u(t) = GKx(t) \tag{9}$$

where

$$G = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{10}$$

Note that Eq. (9) represents a suboptimal controller to Eq. (2) in that the relative position and velocity variables in the x direction are not used. In order to account for the axial force (thrust and drag in the x direction), the controller of Eq. (9) can be modified as

$$u(t) = -GKx(t) - u_m \tag{11}$$

where $u_m = (f_{nx}\ 0\ 0)^T$ respresents a measurable achieved missile acceleration in the x direction. It will be assumed that f_{nx} is a constant, independent of time, although this may not be true for all scenarios.

Both Eqs. (9) and (11) will be studied for esimating tgo via Eq. (7). The resulting tgo found will then be used in the guidance control law based upon Eqs. (5) and (6) developed by Riggs.[5] The closed-loop system matrix using either Eq. (9) or (11) becomes

$$\bar{A} = A - BGK \tag{12}$$

while $c(t)$, the disturbance matrix, is identically zero for Eq. (9) but equal to u_m for Eq (11).

Substitution of Eq. (8) into Eq. (7) assuming Eq. (12) finds

$$J_{\mathrm{tgo}} = x^T(t) e^{\bar{A}^T \mathrm{tgo}} S_f e^{\bar{A}\mathrm{tgo}} x(t) + 2x^T(t) e^{\bar{A}^T \mathrm{tgo}} S_f \int_0^{\mathrm{tgo}} e^{\bar{A}\tau}\mathrm{d}\tau Bc + c^T B^T \int_0^{\mathrm{tgo}} e^{\bar{A}^T \tau}\mathrm{d}\tau S_f \int_0^{\mathrm{tgo}} e^{\bar{A}\tau}\mathrm{d}\tau Bc + q\mathrm{tgo}^2 \tag{13}$$

where $c(t)$ is assumed constant and t corresponds to the present time.

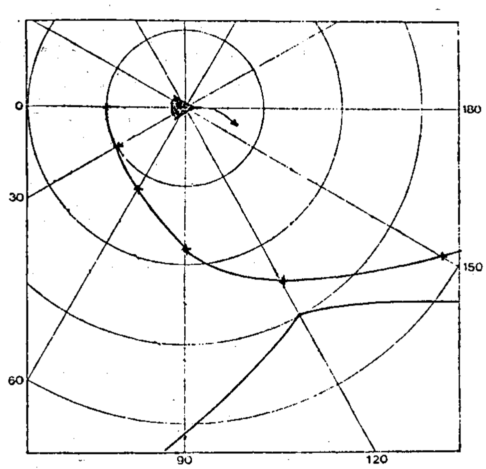

Fig. 2 Inner launch boundary for 40 deg off-bore sight angle (—pronav, – + – guidance law of Ref. 5).

Direct minimization of Eq. (13) with respect to tgo implies equating the following to zero:

$$\begin{aligned}
\frac{\partial}{\partial \mathrm{tgo}} J_{\mathrm{tgo}} &= 2x^T(t)\, e^{\tilde{A}^T \mathrm{tgo}} S_f \frac{\partial}{\partial \mathrm{tgo}} (e^{\tilde{A}\mathrm{tgo}})\, x(t) \\
&\quad + 2x^T(t) \left\{ \frac{\partial}{\partial \mathrm{tgo}} (e^{\tilde{A}^T \mathrm{tgo}}) S_f \int_0^{\mathrm{tgo}} e^{\tilde{A}\tau} \mathrm{d}\tau \right. \\
&\quad \left. + e^{\tilde{A}^T \mathrm{tgo}} S_f \left[e^{\tilde{A}\mathrm{tgo}} + \int_0^{\mathrm{tgo}} \frac{\partial}{\partial \mathrm{tgo}} (e^{\tilde{A}\tau})\, \mathrm{d}\tau \right] \right\} Bc \\
&\quad + 2c^T B^T \int_0^{\mathrm{tgo}} e^{\tilde{A}^T \tau} \mathrm{d}\tau S_f \left[e^{\tilde{A}\mathrm{tgo}} + \int_0^{\mathrm{tgo}} \frac{\partial}{\partial \mathrm{tgo}} (e^{\tilde{A}\tau})\, \mathrm{d}\tau \right] Bc + 2q\mathrm{tgo} \\
&= 2 \left[x^T(t)\, e^{\tilde{A}^T \mathrm{tgo}} S_f \frac{\partial}{\partial \mathrm{tgo}} (e^{\tilde{A}\mathrm{tgo}})\, x(t) \right. \\
&\quad + x^T(t) \frac{\partial}{\partial \mathrm{tgo}} (e^{\tilde{A}^T \mathrm{tgo}}) S_f \int_0^{\mathrm{tgo}} e^{\tilde{A}\tau} \mathrm{d}\tau Bc \\
&\quad + x^T(t)\, e^{\tilde{A}^T \mathrm{tgo}} S_f e^{\tilde{A}\mathrm{tgo}} Bc + x^T(t)\, e^{\tilde{A}^T \mathrm{tgo}} S_f \int_0^{\mathrm{tgo}} \frac{\partial}{\partial \mathrm{tgo}} (e^{\tilde{A}\tau})\, \mathrm{d}\tau Bc \\
&\quad + c^T B^T \int_0^{\mathrm{tgo}} e^{\tilde{A}^T \tau} \mathrm{d}\tau S_f e^{\tilde{A}\mathrm{tgo}} Bc \\
&\quad \left. + c^T B^T \int_0^{\mathrm{tgo}} e^{\tilde{A}^T \tau} \mathrm{d}\tau S_f \int_0^{\mathrm{tgo}} \frac{\partial}{\partial \mathrm{tgo}} (e^{\tilde{A}\tau})\, \mathrm{d}\tau Bc + q\mathrm{tgo} \right]
\end{aligned} \tag{14}$$

The weighting q is normally chosen so that equal importance is placed on each term in Eq. (7). Equating Eq. (14) to zero leads to either a first- or third-order polynomial in tgo that can be solved at each iteration. These equations can be solved in a recursive fashion at each iteration (see Appendix). When $c=0$, this leads directly to the following estimate of tgo (see Appendix):

$$\mathrm{tgo} = -\frac{x_p^T(t) D F x_v(t)}{(x_v^T(t) F^T F x_v(t) + q)} \tag{15}$$

where

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & d & 0 \\ 0 & 0 & d \end{bmatrix}, \quad F = \begin{bmatrix} 1 & 0 & 0 \\ 0 & f & 0 \\ 0 & 0 & f \end{bmatrix}$$

$$d = e^{-3/2}\cos(\sqrt{3}/2) + \sqrt{3}e^{-3/2}\sin(\sqrt{3}/2)$$

and

$$f = (2/\sqrt{3})\, e^{-3/2}\sin(\sqrt{3}/2) \tag{16}$$

IV. Simulation Results

Simulation with various estimation algorithms and the guidance law of Riggs was performed using the six degree-of-freedom simulation package developed by the Eglin Air Force Base Armament Laboratory. The software uses an evasive target algorithm and performs missile-target profiles for user-defined off-bore sight and aspect angle parameters.

Preliminary simulation results illustrated the necessity of a good initial guess of time-to-go. An earlier estimation algorithm developed by Riggs[5] which gave promising results was based upon range R, closing velocity V_c, and off-bore sight angles Bay (angle between missile body acceleration in the x direction and the line-of-sight vector at launch). This estimator was used to generate an initial guess of Eq. (14). In particular, the initial guess was chosen as

$$\mathrm{tgo}(0) = \frac{2R}{V_c + \sqrt{V_c^2 + 52R\cos(\mathrm{Bay})g}} \tag{17}$$

where $g = 32.175$ ft/s.^{2}.

Tests were conducted using Eqs. (9), (11), and (17) with a guidance law developed by Riggs. An off-bore sight angle of 40 deg was chosen in order to compare time-to-go estimation algorithms under complex scenarios. Table 1 shows the inner launch boundaries (minimum launch range in which the missile still hits the target) for various yaw aspect angles (angle between the targets velocity vector and the line-of-sight at launch) using the following time-to-go estimators:

1) Proportional navigation (pro-nav).
2) New G^5: $\mathrm{tgo}(i+1) = 2R/(V_c + \sqrt{V_c^2 + 4\tilde{A}Rg})$, where

$$\tilde{A} = [101.4 - 39t - 12\mathrm{tgo}(i)]/\mathrm{tgo}(i)$$

3) Lee: tgo obtained from Eqs. (14) and (17). The same guidance control law was used in each case.

From these results, one observes that the methodology developed in this paper shows an improvement over pro-nav and equivalent or better than New G at all aspect angles except 150 and 180 deg. Furthermore, the computation required in estimating time-to-go is comparable to these other methods. The influence of adding the integral of the time term in Eq. (7), i.e., $q \neq 0$, results in a slight improvement at an aspect angle of 150 deg. The influence of adding the disturbance term $c = u_m$ results in an improvement over New

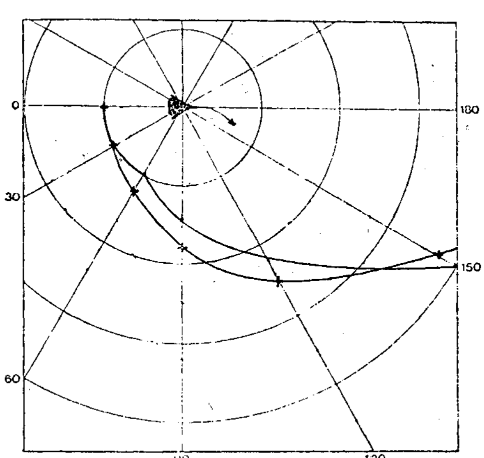

Fig. 3 Inner launch boundary comparison for two guidance control laws, 40 deg off-bore sight angle (—guidance law using Eq. (15), – + – guidance law of Ref. 5).

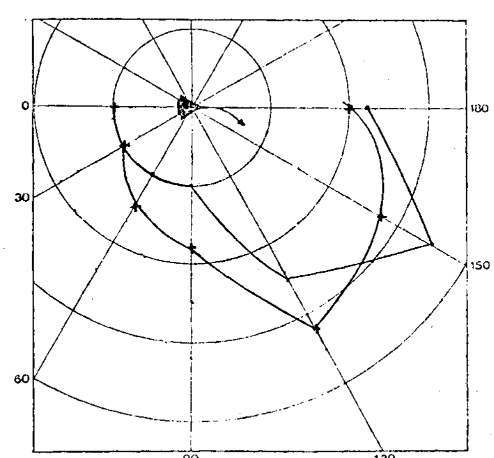

Fig. 4 Inner launch boundary comparison for two guidance laws, 0 deg off-bore sight angle (—guidance law using Eq. (15), – + – guidance law of Ref. 5).

Table 1 Inner launch boundaries

Pro-nav							
Aspect angle, deg	0	30	60	90	120	150	180
Range	–	–	5750	4250	3000	4750	4750
New G							
Aspect angle, deg	0	30	60	90	120	150	180
Range	1000	1000	1250	1750	2500	3750	3750
Lee ($q \neq 0, c=0$)							
Aspect angle, deg	0	30	60	90	120	150	180
Range	1000	1000	1000	1500	2250	4500	4500
Lee ($q=0, c \doteq u_m$)							
Aspect angle, deg	0	30	90	120	120	150	180
Range	1000	1000	1000	1500	2250	4000	4000
Lee ($q=0, c=u_m$)							
Aspect angle, deg	0	30	60	90	120	150	180
Range	1000	1000	1000	1500	2250	4250	4500
Lee ($q \neq 0, c-u_m$)							
Aspect angle, deg	0	30	60	90	120	150	180
Range	1000	1000	1000	1500	2250	3750	4000

Table 2 Inner launch boundaries for 0 deg off-bore sight

Pro-nav							
Aspect angle, deg	0	30	60	90	120	150	180
Range	1000	1000	1750	2000	3500	3500	2250
New G							
Aspect angle, deg	0	30	60	90	120	150	180
Range	1000	1000	1500	1750	3250	2750	2000
Lee ($q \neq 0, c=0$)							
Aspect angle, deg	0	30	60	90	120	150	180
Range	1000	1000	1000	1000	2500	3500	2250

G at the higher aspect angles. However, the amount of computation is much more than with $c=0$, which gave the most appealing algorithm for estimating time-to-go, i.e., the algorithm described by Eq. (15). One may observe the estimator philosophies behind New G and Lee and why these estimator algorithms achieve good results. The estimator New G assumes that the acceleration along the line of sight has, as its dominant factor, the achieved missile body acceleration in the x direction, i.e., $\tilde{A}$. Hence, the dominant parameter in estimating tgo using New G is the missile's achieved x-direction acceleration. However, to compensate for lack of availability of commanded x-direction control, the methodology developed in this paper uses the influence of lateral and longitudinal commanded controls [through $x(t_f)$]. In either case, the algorithms compensate for the unavailable command in the x direction and the assumption of zero target acceleration.

The methodology was also used on a simple 0 deg off-bore sight scenario. Table 2 lists the results along with those of pro-nav and New G.

Figures 2-4 illustrate the results of simulation studies for the various time-to-go estimator algorithms with the guidance laws of Ref. 5.

V. Conclusion

An algorithm for estimating time-to-go has been developed using an objective function consisting of terminal miss distance and time. It was found that the methodology gave promising results under minimal computational burden. By constructing and optimizing the objective functions, one is able to observe iteratively how time-to-go changes with respect to current miss distance and elapsed time. Several areas of study need still be investigated:

1) The development of tgo was based under the assumption that the target acceleration was zero. As this is not true, the system dynamics need to be modified, which in turn alters the time-to-go estimate.

2) The constraint on the missile's commanded controls was set to a maximum value above current missile capabilities in the simulation program. Hence the missile's command controls found via the time-to-go estimate were above realistic values. By lowering the maximum acceleration level, a degradation in system performance was observed using the current tgo estimator. To improve performance under these missile capabilities, work needs to be done in modifying the weighting factor on time. Presently, as time and terminal miss distance are weighted equally, the missile tries to use maximum acceleration values to achieve minimal time.

Appendix

In order to evaluate the coefficients of time-to-go (tgo) in Eq. (14), it is necessary to compute the following terms:

$$e^{\tilde{A}\mathrm{tgo}}, \int_0^{\mathrm{tgo}} e^{\tilde{A}t}\mathrm{d}t, \text{ and } \int_0^{\mathrm{tgo}} \frac{\partial}{\partial \mathrm{tgo}}(e^{\tilde{A}t})\mathrm{d}t.$$

The matrix $e^{\tilde{A}t}$ can be computed directly using Laplace transforms as

$$e^{\tilde{A}t} = \begin{bmatrix} 1 & 0 & 0 & t & 0 & 0 \\ 0 & \alpha_1(t) & 0 & 0 & \alpha_2(t) & 0 \\ 0 & 0 & \alpha_1(t) & 0 & 0 & \alpha_2(t) \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & \alpha_3(t) & 0 & 0 & \alpha_4(t) & 0 \\ 0 & 0 & \alpha_3(t) & 0 & 0 & \alpha_4(t) \end{bmatrix} \quad \text{(A1)}$$

where

$$\alpha_1(t)=e^{-(3/2\text{tgo})t}\cos\left(\frac{\sqrt{3}}{2\text{tgo}}\right)t+\sqrt{3}e^{-(3/2\text{tgo})t}\sin\left(\frac{\sqrt{3}}{2\text{tgo}}\right)t$$

$$\alpha_2(t)=\frac{2\text{tgo}}{\sqrt{3}}e^{-(3/2\text{tgo})t}\sin\left(\frac{\sqrt{3}}{2\text{tgo}}\right)t$$

$$\alpha_3(t)=\frac{2\sqrt{3}}{\text{tgo}}e^{-(3/2\text{tgo})t}\sin\left(\frac{\sqrt{3}}{2\text{tgo}}\right)t$$

and

$$\alpha_4(t)=e^{-(3/2\text{tgo})t}\cos\left(\frac{\sqrt{3}}{2\text{tgo}}\right)t-\sqrt{3}e^{-(3/2\text{tgo})t}\sin\left(\frac{\sqrt{3}}{2\text{tgo}}\right)t \tag{A2}$$

The matrix can also be computed indirectly through a truncated series

$$e^{\bar{A}\text{tgo}}=\begin{bmatrix}\Sigma\dfrac{A_{11}(i)}{i!} & \Sigma\dfrac{A_{12}(i)}{i!}\text{tgo}\\ \Sigma\dfrac{A_{21}(i)}{i!\text{tgo}} & \Sigma\dfrac{A_{22}(i)}{i!}\end{bmatrix} \tag{A3}$$

where

$$A_{11}(i)=-3G[A_{11}(i-2)+A_{11}(i-1)]$$

$$A_{12}(i)=-3G[A_{12}(i-2)+A_{12}(i-1)]$$

$$A_{21}(i)=3G[2A_{11}(i-1)+3A_{11}(i-2)]$$

$$A_{22}(i)=3G[2A_{12}(i-1)+3A_{12}(i-2)] \qquad i=3,4,\ldots \tag{A4}$$

and

$$\begin{array}{lll} A_{11}(0)=I & A_{11}(1)=0 & A_{11}(2)=-3G\\ A_{12}(0)=0 & A_{12}(1)=I & A_{12}(2)=-3G\\ A_{21}(0)=0 & A_{21}(1)=-3G & A_{21}(2)=9G^2\\ A_{22}(0)=I & A_{22}(1)=-3G & A_{22}(2)=9G^2-3G \end{array} \tag{A5}$$

With $c=u_m$, the matrices

$$\int_0^{\text{tgo}} e^{\bar{A}t}\mathrm{d}t \text{ and } \int_0^{\text{tgo}} \frac{\partial}{\partial\text{tgo}}(e^{\bar{A}t})\mathrm{d}t$$

also need to be computed, which adds to further computation. Direct computation analytically using Eq. (1) requires integration by parts. Hence, for $c=u_m$, using Eq. (A3) is more attractive. It was found that

$$\int_0^{\text{tgo}} e^{\bar{A}\tau}\mathrm{d}\tau=\begin{bmatrix}\Sigma\dfrac{V_{11}(i)}{i!}\text{tgo} & \Sigma\dfrac{V_{12}(i)}{i!}\text{tgo}^2\\ \Sigma\,\pi\dfrac{V_{12}(i)}{i!} & \Sigma\dfrac{V_{22}(i)}{i!}\end{bmatrix} \tag{A6}$$

where $V_{jk}(i)$ has the same form as Eq. (A4) with the initial conditions

$$V_{jk}(0)=0$$

$$V_{jk}(1)=A_{jk}(0)$$

$$V_{jk}(2)=A_{jk}(1) \tag{A7}$$

Finally

$$\int_0^{\text{tgo}} \frac{\partial}{\partial\text{tgo}}(e^{\bar{A}t})\mathrm{d}t=\begin{bmatrix}\Sigma\dfrac{W_{11}(i)}{(i+1)!} & \Sigma\dfrac{W_{12}(i)}{(i+1)!}\text{tgo}\\ \Sigma\dfrac{W_{21}(i)}{(i+1)!\text{tgo}} & \Sigma\dfrac{W_{22}(i)}{(i+1)!}\end{bmatrix} \tag{A8}$$

where

$$W_{11}(i)=6GA_{11}(i-2)-3GW_{11}(i-2)+3GA_{11}(i-1)-3GW_{11}(i-1)$$

$$W_{12}(i)=6GA_{12}(i-2)-3GW_{12}(i-2)+3GA_{12}(i-1)-3GW_{12}(i-1)$$

$$W_{21}(i)=-27G^2A_{11}(i-2)+9G^2W_{11}(i-2)-18G^2A_{11}(i-1)+9G^2W_{11}(i-1)+6GA_{11}(i-1)-3GW_{11}(i-1)$$

$$W_{22}(i)=-27G^2A_{12}(i-2)+9G^2W_{12}(i-2)-18G^2A_{12}(i-1)+9G^2W_{12}(i-1)+6GA_{12}(i-1)-3GW_{12}(i-1) \tag{A9}$$

and

$$W_{jk}(0)=W_{jk}(1)=0$$

$$\begin{array}{ll} W_{11}(2)=6G & W_{12}(2)=3G\\ W_{21}=-27G^2 & W_{22}(2)=-18G^2+6G \end{array}$$

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